Int. J. Heat Mass Transfer. Vol. 11, pp. 769–772. Pergamon Press 1968. Printed in Great Britain

SHORTER COMMUNICATIONS

A LIQUID METAL HEAT-TRANSFER EXPERIMENT AND ITS RELATION TO RECENT THEORY*

RICHARD L. MERRIAM†, RALPH P. STEIN‡ and BOBBIE L. RICHARDSON§

(*Received* 20 *July* 1967 *and in revised form* 9 *October* 1967)

NOMENCLATURE

c_i, heat capacity of fluid in channel i;
D_1, inside diameter of tube;
D_2, inside diameter of annular space;
D_3, outside diameter of annular space;
F, eigenvalue function, equation (4);
H, heat capacity flow rate ratio, $c_2 W_2/c_1 W_1$;
J_n, Bessel functions of first kind;
k_i, thermal conductivity of fluid in channel i;
k_i^+, thermal conductivity multiplier for turbulent flow in channel i;
k_w, thermal conductivity of wall;
K, fluid thermal resistance ratio, $(k_1k_1^+/k_2k_2^+)(1 - R/R)$;
K_w, relative wall thermal resistance, $(k_1k_1^+/k_w)\ln(D_2/D_1)$;
l, axial distance:
M_n, special Bessel funtion, equation (7);
$Nu_i(\infty)$, fully developed Nusselt number for channel i;
$[Nu]_p$, "plug flow" Nusselt number;
Pe_1, Péclet number in channel 1, $4c_1 W_1/k_1\pi D_1$;
R, annular ratio, D_2/D_3;
W_i, mass flow rate of fluid in channel i;
Y_n, Bessel functions of second kind;
z, dimensionless axial distance, $(4k_1^+/Pe_1)(l/D_1)$.

Greek symbols
ε, heat exchanger efficiency;
λ, eigenvalues, equation (4);
ϕ, effectiveness coefficient, equation (6);
ω^2, $(R/1 + R)KH$.

Subscripts
1, refers to tube side;
2, refers to annular side.

* Based on an M.S. thesis submitted by the senior author to Marquette University.

† Graduate Assistant, School of Mechanical Engineering, Purdue University, West Lafayette, Indiana 47907, U.S.A.

‡ Reactor Engineering Division, Argonne National Laboratory, Argonne, Illinois 60439, U.S.A.

§ Chairman, Department of Mechanical Engineering, Marquette University, Milwaukee, Wisconsin 53233, U.S.A.

RECENT analyses [1, 2, 3] pertaining to liquid metal heat transfer in double pipe heat exchangers have indicated that significant inaccuracies in predictions of overall heat-transfer rates and in interpretations of experimental data—especially as related to heat-transfer coefficients—can result when based upon traditional heat exchanger design relations. The main purpose of the investigation of [4] was to verify these analytical results experimentally for turbulent cocurrent flow mercury–mercury heat exchangers. This note summarizes some of the more important results of this investigation. Mathematical and experimental details are given in [4].

ANALYSIS

In the interest of brevity only the main working equations used to calculate the analytical predictions are presented. The derivation of these equations follows the procedure outlined in [3] with the additional assumption that the velocity distributions within the fluids are uniform. The effect of eddy transport on the heat transfer in the fluids is accounted for by a quantity k_i^+ called "the thermal conductivity multiplier for turbulent flow". Values of this quantity were computed from the correlations for heat transfer in a tube and in an annulus with a uniform heat flux boundary condition [6, 8] and from the corresponding plug flow values [9],

$$Nu_i = k_i^+[Nu_i]_p. \quad (1)$$

Justification of this procedure is given in [2].

The resulting equations for the tube and annulus side Nusselt numbers are

$$Nu_1(\infty) = k_i^+ \frac{2\lambda_1^2 J_1(\lambda_1)}{2J_1(\lambda_1) - \lambda_1 J_0(\lambda_1)}, \quad (2)$$

$$Nu_2(\infty) = k_2^+ \frac{2\omega^2\lambda_1^2 M_1\left(\frac{\omega R}{1-R}\lambda_1\right)}{\frac{2R}{1+R}M_1\left(\frac{R}{1-R}\lambda_1\right) + \omega\lambda_1 M_0\left(\frac{R}{1-R}\lambda_1\right)}, \quad (3)$$

where the quantity λ_1 is the smallest non-zero eigenvalue satisfying

$$F(\lambda) = K_w \lambda J_1(\lambda) M_1\left(\frac{\omega R}{1-R}\lambda\right) - J_0(\lambda) M_1\left(\frac{\omega R}{1-R}\lambda\right) + \frac{K}{\omega} J_1(\lambda) M_0\left(\frac{\omega R}{1-R}\lambda\right) = 0. \quad (4)$$

Values for the heat exchanger "efficiency" were computed from

$$\varepsilon(z) = 1 - \phi(z)\exp(-\lambda_1^2 z) \quad (5)$$

where

$$\phi(z) = \frac{1+H}{H}\sum_{n=1}^{\infty} \frac{4M_1\left(\frac{\omega R}{1-R}\lambda_n\right) J_1(\lambda_n)}{\lambda_n^2 F'(\lambda_n)} \times \exp[-(\lambda_n^2 - \lambda_1^2)z]. \quad (6)$$

The quantities M_n are special Bessel functions defined by

$$M_n\left(\frac{\omega R}{1-R}\lambda\right) = J_n\left(\frac{\omega R}{1-R}\lambda\right) Y_1\left(\frac{\omega}{1-R}\lambda\right) - J_1\left(\frac{\omega}{1-R}\lambda\right) Y_n\left(\frac{\omega R}{1-R}\lambda\right). \quad (7)$$

EXPERIMENTS

Experiments were performed with simple cocurrent flow double pipe heat exchangers of various lengths. The results obtained with the longest section, in which fully developed heat transfer was attained are reported here.

The test section, illustrated in Fig. 1, was constructed of nickel and was treated by a 50 per cent solution of hydrochloric acid just prior to charging the flow loop to enhance thermal "wetting". An insulating air gap provided a hydraulic entrance region prior to the heat-transfer region.

Bulk fluid temperature measurements were made at the entrance and exit of the test section to determine overall heat-transfer rates. A thermocouple imbedded in the inner tube wall gave measurements of the wall temperature at the outlet end of the exchanger. Since the temperature change through the inner tube wall was relatively small, the heat-transfer surface temperatures could be determined quite accurately once the local heat flux was known. The local heat flux at the outlet end of the exchanger was determined from detailed measurements of the outer wall temperature distribution by the method developed by Stein [2, 3, 5]. The method also provides a test to determine if the heat transfer is fully developed.

FULLY DEVELOPED NUSSELT NUMBERS

With the exit bulk fluid temperatures, heat-transfer surface temperatures, and local heat flux known, the individual channel fully developed heat-transfer coefficients could be computed. Figure 2 shows a comparison of the tube side fully developed Nusselt number with the various analytical predictions for flow with a fixed Péclet number of 100 in the annulus. The Lyon equation [6] and the Seban–

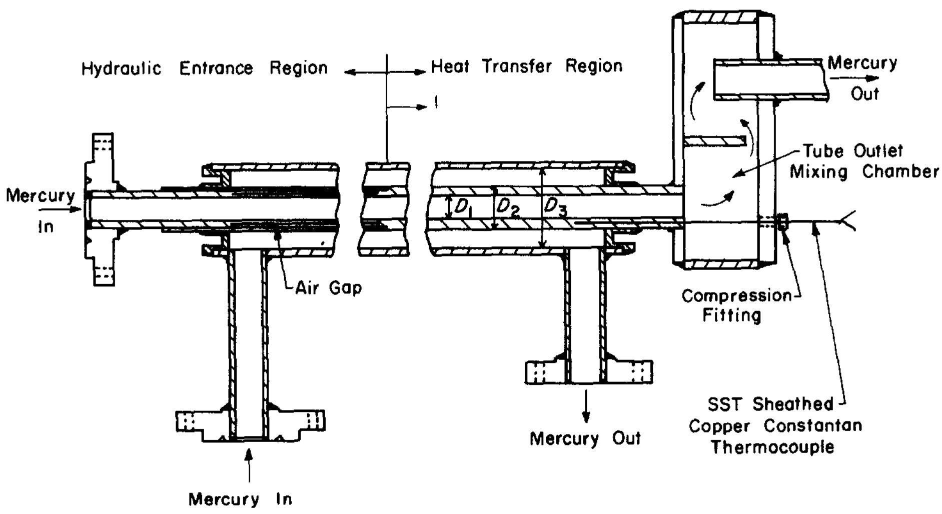

FIG. 1. Test section design.

Shimazaki equation [7] were used to predict the tube side Nusselt numbers appropriate to the uniform heat flux and uniform surface temperature boundary conditions respectively. In usual practice it is assumed that these two boundary conditions represent the extreme cases. The experimental results, in agreement with the predictions of the analysis of [4], indicate that fully developed heat-transfer coefficients may be actually lower than those predicted for a uniform surface temperature boundary condition. Figure 3 shows a similar comparison for the annular side Nusselt number for the same operating conditions of Fig. 2. Note that although the annular side Péclet number is constant, the annular side Nusselt number increases with increasing *tube side* Péclet number as predicted by the analysis. The Nusselt numbers for the boundary conditions of uniform wall temperature and uniform heat flux were obtained from the analysis of [4] and the correlations of Dwyer [8] respectively.

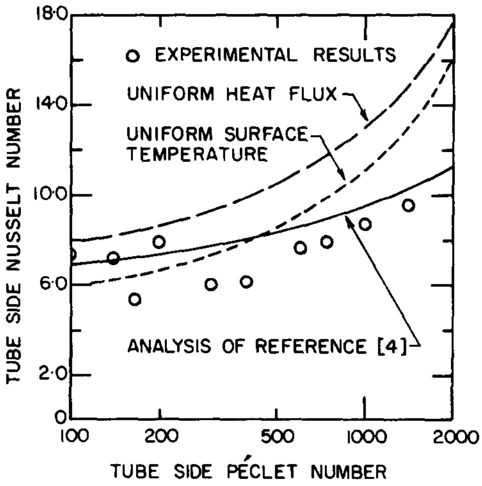

FIG. 2. Fully developed tube side Nusselt number vs. tube side Péclet number (annular side Péclet number = 100).

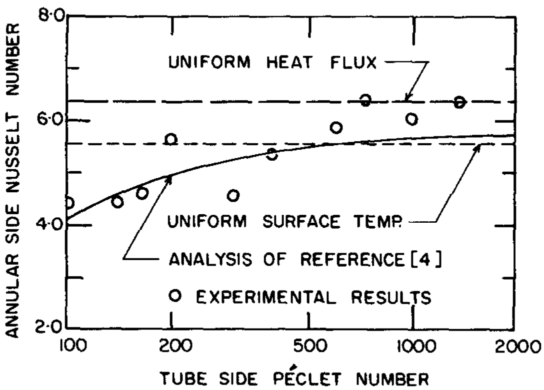

FIG. 3. Fully developed annular side Nusselt number vs. tube side Péclet number (annular side Péclet number = 100).

OVERALL HEAT-TRANSFER RATES

The importance of the actual heat-transfer boundary conditions is further illustrated in Fig. 4 where results for heat exchanger "efficiency" are presented for a fixed flow rate in the annulus. "Efficiency" is taken here to mean the ratio of the actual heat-transfer rate to the rate that would be obtained in an infinitely long heat exchanger operating under the same conditions. Customary heat exchanger design methods are based on the assumption that the heat-transfer coefficients are uniform along the length of the exchanger. When heat-transfer boundary conditions are important, as with liquid metals, traditional design methods imply use of heat-transfer coefficients corresponding to the uniform surface temperature condition as the most appropriate to a cocurrent flow heat exchanger. The line labeled "see text" represents predictions according to the customary design equation using fully developed heat-transfer coefficients appropriate to a uniform heat flux boundary condition.

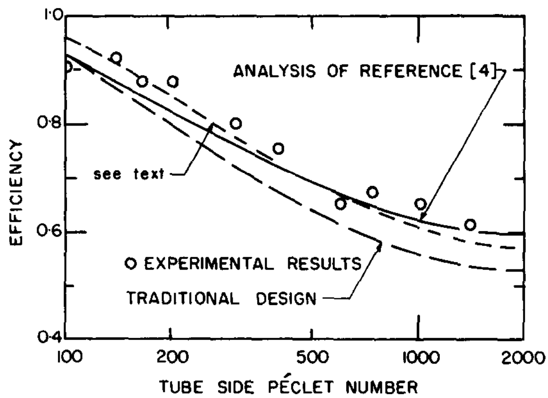

FIG. 4. Heat exchanger efficiency vs. tube side Péclet number (annular side Péclet number = 100).

The traditional design method predicts the lowest value of the efficiencies shown. The experimental results agree equally well with the analysis of [4] and with the predictions based upon the assumption of a uniform heat flux. Use of this assumption tends to correct for the high rates of heat-transfer in the thermal entrance regions which are not otherwise accounted for by the customary analysis. As shown in [4], for efficiencies less than 0·5 calculations based on the assumption of either uniform heat flux or uniform surface temperature boundary conditions may be seriously in error.

ACKNOWLEDGEMENTS

The work on this investigation was supported by the United States Atomic Energy Commision, Division of Reactor Development, Engineering Development Branch, and was carried out at Argonne National Laboratory, Reactor Engineering Division.

REFERENCES

1. R. P. STEIN, The Graetz problem in concurrent double pipe heat exchangers, *Chem. Engng Prog. Symp. Ser.* **61**, 76 (1965).
2. R. P. STEIN, Liquid metal heat transfer, in *Adv. Heat Transf.*, edited by T. F. IRVINE JR. and J. P. HARTNETT, Vol. 3, p. 101. Academic Press, New York (1966).
3. R. P. STEIN, Mathematical and practical aspects of heat transfer in double pipe heat exchangers, in *Proceedings of the Third International Heat Transfer Conference*, Vol. 1, p. 139. Am. Inst. Chem. Engrs, New York (1966).
4. R. L. MERRIAM, An investigation of liquid–metal heat transfer in a concurrent-flow, double pipe heat exchanger, ANL-7056 (1965).
5. R. P. STEIN, A method for the determination of local heat fluxes in liquid metal heat exchangers, in Proceedings of the Conference on Application of High-Temperature Instrumentation to Liquid–Metal Experiments, 28–29 September 1965, 483, ANL-7100 (1966).
6. R. N. LYON, Liquid metal heat transfer coefficients, *Chem. Engng Prog.* **47**, 75 (1951).
7. R. A. SEBAN and T. T. SHIMAZAKI, Heat transfer to a fluid flowing turbulently in a smooth pipe with walls at constant temperatures, *Trans. Am. Soc. Mech. Engrs* **73**, 803 (1951).
8. O. E. DWYER, On the transfer of heat to fluids flowing through pipes, annuli, and parallel plates, *Nucl. Sci. Engng*, **17**, 336 (1963).
9. L. M. TREFETHEN, Liquid metal heat transfer in circular tubes and annuli, *General Discussion on Heat Transfer*, p. 43. Inst. Mech. Engrs–Am. Soc. Mech. Engrs, New York (1951).

Int. J. Heat Mass Transfer. Vol 11, pp. 772–774. Pergamon Press 1968. Printed in Great Britain

THE EFFECTIVENESS OF A COUNTER-FLOW HEAT EXCHANGER WITH CROSS-FLOW HEADERS

W. M. KAYS,* R. K. JAIN† and S. SABHERWAL†

(*Received* 6 *July* 1967 *and in revised form* 6 *November* 1967)

NOMENCLATURE

A, heat-transfer surface area on either hot or cold fluid sides;

C, fluid capacity rate ($= \dot{m}c_p$);

c_p, fluid specific heat;

C_{min}/C_{max}, capacity rate ratio;

E, heat exchanger effectiveness,

$$E = \frac{C_h(t_{h,in} - t_{h,out})}{C_{min}(t_{h,in} - t_{c,in})} = \frac{C_c(t_{c,out} - t_{c,in})}{C_{min}(t_{h,in} - t_{c,in})};$$

$\dot{m}$, fluid mass flow-rate;

N_{tu}, number of transfer units, $N_{tu} = (AU/C_{min})$; Note: A and U must be based on the same side of the heat exchanger; the product (AU) is the same on both the hot and cold sides;

U, overall unit heat-transfer conductance from hot side fluid to cold side fluid, based on a unit of area A.

Subscripts

c, "cold" fluid side;

h, "hot" fluid side;

min, the side with the minimum of either C_c or C_h.

A DIFFICULTY in building a counter-flow heat exchanger using very compact plate-fin heat-transfer surfaces arises in the separation of the two fluids at the ends of the heat exchanger. Some kind of cross-flow sandwich header must be used, and the usual procedure is to extend the plates to form one or other of the two patterns shown in Fig. 1. Then the fins in the end sections are brazed in such a manner as to turn the flow and separate the fluids as in a cross-flow heat exchanger.

The result is a composite heat exchanger with a true counter-flow central core, but with cross-flow headers. One can anticipate that the performance, based on the *total* heat-transfer surface area, would be inferior to that of a

* Professor of Mechanical Engineering, Stanford University, Stanford, California.

† Graduate students, Mechanical Engineering, Imperial College of Science and Technology, London.